Topology III - Semestral Examination, MMath II.

Max. Marks : 60 Time : 3 hours

Answer all questions. You may use theorems/propositions proved in the class after correctly stating them. Any other claim must be accompanied by a proof.

(1) (a) Compute the groups $H^i(X;G)$ where $X = S^2 \times \mathbb{R}P^2$ and $G = \mathbb{Z}, \mathbb{Z}_2$. [6+6]

(b) Compute the cohomology ring $H^*(\mathbb{C}P^n;\mathbb{Z})$. [10]

(c) Let X be a compact orientable 5-manifold. Say as much as you can about the homology groups $H_i(X;\mathbb{Z})$. [10]

(2) (a) Define the unreduced suspension SX of a space X. Show that SS^n is homeomorphic to S^{n+1}. [2+4]

(b) Define the term *fibration*. Let $E = \{(x, y) \in \mathbb{R}^2 : |y| \leq |x|\}$. Show that the map $p : E \longrightarrow \mathbb{R}$ defined by $p(x, y) = x$ is a fibration. Explain why p cannot be a fiber bundle. [2+6+2]

(c) Show that the quotient map $p : S^{2n+1} \longrightarrow \mathbb{C}P^n$ is a fiber bundle with fiber S^1. Show that the quotient map $p : S^3 \longrightarrow \mathbb{C}P^1 = S^2$ is essential. [6+6]